Enhancement of Photospheric Meridional Flow by Reconnection Processes

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ABSTRACT

We simulate the two-dimensional transport of the open magnetic flux on the surface of the Sun. The temporal evolution of the flux density depends on the advective motions due to solar differential rotation and poleward meridional flow, and on an effective spatial diffusive motion. The latter is a result of the uniform diffusion of field lines footpoints in the network lanes, and a non-uniform diffusion of field lines due to reconnection of open field lines with closed loops on the solar surface. The gradient of the diffusion coefficient represents an effective velocity in addition to the advective velocity. We investigate the behavior of the steady state solution for solar minimum and solar maximum conditions with spatially uniform and non-uniform diffusion coefficients. We find that for solar minimum conditions, the effect of spatial diffusion resulting from reconnection processes enhances the poleward meridional flow due to the large-scale preferred direction in the gradient of the diffusion coefficient. For solar maximum conditions, the net effect of spatial diffusion is minimal because of the isotropic and local gradients of the diffusion coefficient. Our simulation demonstrates that magnetic reconnection processes on the solar surface can be a mechanism to vary motions on the photosphere, in particular, poleward meridional flow.

Subject headings: MHD — Sun: corona — Sun: magnetic field — Sun: photosphere — Sun: activity

1. Introduction

Meridional flow is significant both in the photospheric flux-transport models (Wang et al. 1989, 2002, 2000a,b, 2005; Mackay et al. 2002a,b), and in flux-transport dynamo models

(Dikpati et al. 2004; Dikpati 2005). It has been demonstrated by both types of models that an average meridional flow with a magnitude on the order of $10-20\ m\ s^{-1}$ is required to maintain the magnetic dynamo, and to obtain the field reversal. In addition, some recent papers suggest that a modification of the meridional flow can explain the variations from one solar cycle to another (Schrijver et al. 2002; Dikpati et al. 2004; Wang et al. 2005).

The surface motions on the photosphere consist of differential rotation, random motions, and meridional flow. While the differential rotation has been extensively investigated using GONG and SOHO MDI measurements (Hathaway 1996; Hathaway et al. 1996; Schou et al. 1998), and the random motion has been studied by several authors (Leighton 1964; Wang et al. 1989; Schrijver et al. 1996; Cameron and Hopkins 1998), it is difficult to characterize the meridional flow due to its weak signal compared to the differential motion. Early estimations of the meridional flow from day-to-day ground based Doppler measurements (Duvall 1979) obtained a flow of the order of $20\ m\ s^{-1}$. More recent Doppler estimations were done using GONG and MDI data (Hathaway 1996; Basu and Antia 2003), and were in agreement with an average flow of $20\ m\ s^{-1}$. However, these measurements showed that the flow can be enhanced by a factor of 2-3 during the solar cycle. Helioseismology analysis of MDI data (Zhao and Kosovichev 2004) obtained a meridional flow on the order of $20\ m\ s^{-1}$ as well. A different approach to estimate the photospheric meridional flow was done by tracking the motions of magnetic features on the solar surface using magnetogram data (Komm et al. 1996; Latushko 1994; Snodgrass and Dailey 1995). The average meridional flow was found to be of the order of $10\ m\ s^{-1}$ with variations of up to $5\ m\ s^{-1}$ during the solar cycle. All the above observations agree with the theoretical restriction of a maximum meridional flow speed on the order of $10-20\ m\ s^{-1}$.

As mention above, the meridional flow is important for the transport of magnetic flux on the Sun. The magnetic flux of the Sun is composed of closed magnetic flux regions, where both ends of the magnetic field lines are attached to the Sun, and open magnetic flux regions, where one end of the field lines is attached to the Sun and the other end is dragged by the solar wind into the heliosphere. While the closed magnetic field lines are located within the Alfvénic point, the heliospheric magnetic field is controlled by the open magnetic flux. During solar minimum conditions, the open flux geometry is of two coronal holes with opposite polarity separated by a single current sheet; the magnetic polarity is reversed approximately every 11 years, possibly due to rotation of the current sheet (Saito et al. 1978; Fisk and Schwadron 2001). It has been observed that the open magnetic flux of the Sun remains virtually constant through the solar cycle (Wang et al. 2000a, 2002). In addition, it has been argued that CMEs do not change the background amount of open flux through the process of interchange reconnection (Crooker et al. 2002; Fisk 2005). Under these assumptions, Fisk and co-workers, in a series of papers (Fisk 1996, 2005; Fisk et al.

1999; Fisk and Schwadron 2001), constructed a new transport model for the evolution of the open magnetic flux of the Sun. The model proposed by Fisk and co-workers (hereby Fisk model) is an alternative for the common use of the potential field approximation (Altschuler et al. 1977) in describing the solar magnetic field. The potential field approximation assumes that there are no currents in the steady state solar corona, i.e.,

$$\nabla \times \mathbf{B} = 0, \tag{1}$$

and that the magnetic field can be obtained by solving a Laplace's equation for some scalar potential, ψ, where

$$\mathbf{B} = -\nabla\psi. \tag{2}$$

The inner boundary condition for $\mathbf{B}$ is applied through the observed magnetograms on the solar surface and the outer boundary condition assumes that all field lines are radial above the 'source-surface' location (where $\psi = 0$), which is usually set to be at a radial distance of 2.5 solar radii. The potential field approximation is a very convenient tool to describe a static configuration of the solar corona magnetic field and to get the location of the boundary between the open and closed magnetic flux regions. It is also very straightforward to implement in numerical models since it requires only the harmonic coefficient extracted from the measured line-of-sight photospheric magnetic field and the use of associated Legendre polynomials. However, the potential field approximation can only give a static snapshot in time and does not describe the continuous *dynamic* evolution of the coronal magnetic field.

The Fisk model, in turn, proposes a *dynamic* model for the evolution of the open magnetic flux in the photosphere, solar corona, and heliosphere through solar differential rotation and magnetic reconnection of open field lines with closed loops. The effect of solar differential rotation on the evolution of the heliospheric magnetic field as predicted by the model has been studied both theoretically (Schulz 2001; Crooker et al. 2002; Burger et al. 2002, 2004) and numerically (Lionello et al. 2005). In this paper, we present a numerical simulation of the transport of open magnetic flux on the surface of the Sun. We solved the two-dimensional advection-diffusion equation for the open (radial) component of the solar magnetic field taken from Fisk (2005):

$$\frac{\partial B_o}{\partial t} = \nabla^2(\kappa B_o) - \nabla \cdot (\mathbf{u} B_o). \tag{3}$$

The advection term in the equation represents the real motion of the plasma on the solar surface and the components of $\mathbf{u}$ are the solar differential rotation and uniform meridional flow. The diffusion term represents two diffusion processes: (1) diffusion of field lines footpoints in the network lanes and (2) diffusion due to reconnection of open field lines with closed loops proposed by Fisk (2005). We investigate the steady state solution for the photospheric magnetic field for solar minimum and solar maximum conditions. We then compare the results for a case in which we use only the uniform network lanes diffusion, and for another case, in which we include the diffusion due to reconnection. Previous work has been done to study the evolution of the open magnetic flux on the solar surface (Wang et al. 2000a,b, 2002, 2005; Mackay et al. 2002a,b). However, these studies did not include the effect of magnetic reconnection in the diffusion term as our model does. Flux transport through reconnection process can be described as a modification of the large-scale advection motion (Fisk 2005). Therefore, diffusion due to magnetic reconnection can be an effective mechanism to modify the surface flow on the Sun, in particular, the meridional flow which has been observed to have temporal variations. We investigated the role of diffusion resulting from magnetic reconnection in the temporal variations of the meridional flow.

We present the theoretical and numerical model in Section 2. The simulation results are discussed in Sections 3. In Section 4 we draw the main conclusions and discuss future numerical studies.

2. Physical Model

2.1. Theory

The theoretical description of the model for the transport of open magnetic flux due to reconnection with loops follows Fisk (2005). There are several assumptions that the model makes. As mentioned in the previous section, it is assumed that the open magnetic flux remains virtually unchanged during the solar cycle. The size of the loops is much smaller than the size of supergranules, and the loops are randomly oriented. The magnetic flux concentrations on the surface act like particles, so that the random motion of the field line footpoints in the network lanes as well as the motion due to magnetic reconnection can be described as a diffusive process. Every time two field lines with opposite polarity meet each other in the low corona they reconnect. When two loops meet, they reconnect to create a larger loop and a smaller loop of secondary importance. When an open filed line meets a loop, the magnetic reconnection causes the open field line to be transported by a distance of the scale of the size of the loop. In both cases, a smaller loop is created and the model assumes that this loop subducts and gets out of the system.

The mathematical derivation of the model takes into account the rate of emergence of loops, average size, and average amount of magnetic flux of the loops on the solar surface. In addition, some numerical factors arise due to the fact that the diffusion coefficient, κ, is a two-dimensional tensor, and also because magnetic reconnection occurs only on one side of the loop.

The theory manifests in equation (3). The components of the advective velocity $\mathbf{u} = (u_\theta, u_\phi)$ are (Wang et al. 2000b, 2005):

$$u_\phi = 13.38 - 2.30 \sin^2 L - 1.62 \sin^4 L \;\; deg\ day^{-1},$$

which represents the solar differential rotation and,

$$u_\theta = v_m \cos^2 L |\sin L|^{0.025},$$

with

$$v_m = 10 \;\; m\ s^{-1},$$

which represents the poleward meridional flow. Here L is the heliospheric latitude.

The diffusion coefficient, κ, is given by:

$$\kappa = \frac{\delta \bar{h}^2}{2\delta t}\left(1 + \frac{B_l}{B_o}\right). \tag{4}$$

The term $\delta \bar{h}^2/(2\delta t)$ represents the constant diffusion coefficient due to the random motion of the field line footpoints in the network lanes with a typical value of 600 $km^2\ s^{-1}$ (Wang et al. 2000a; Fisk and Schwadron 2001). This diffusion coefficient is modified by the magnetic reconnection processes, and it depends on the density of the open magnetic flux, B_o, and on the magnetic flux contained in the loops emerging on the surface, B_l.

Equation (3) can be written in the following convenient form:

$$\frac{\partial B_o}{\partial t} = -\nabla \cdot \left[(\mathbf{u} + \mathbf{w}) B_o\right], \tag{5}$$

where

$$\mathbf{w} = -\frac{1}{B_o}\nabla\left(\frac{\delta \bar{h}^2}{2\delta t} B_l\right). \tag{6}$$

Here $\mathbf{w}$ is the additional velocity due to the reconnection events. This velocity depends on the gradient of B_l. Therefore, if the rate of emergence of loops is smaller at the poles, there will be an effective poleward motion in addition to the meridional flow. In our simulation, we introduced the following linear dependence of B_l on B_o:

$$B_l = 10 - 8.5 \frac{|B_o|}{\max |B_o|} \quad [G]. \tag{7}$$

Equation (7) states that the magnetic flux of emerging loops B_l is strong (about 10 G) in regions where the magnitude of B_o is weak (outside of coronal holes) and weak (about 1.5 G) in regions where the magnitude of B_o is strong (inside coronal holes) (Fisk 2005).

In general, equations (5,6) describe a mechanism to modify the convective flow ($\mathbf{u}$) on the solar surface. The azimuthal component of $\mathbf{w}$ modifies the differential rotation (u_ϕ), while the meridional component of $\mathbf{w}$ modifies the large-scale surface meridional flow (u_θ). Equation (5) demonstrates that the global motion on the photosphere is a combination of both advective and diffusive motions.

2.2. Grid and Boundary Conditions

For our simulation, we used a uniform, two-dimensional spherical mesh of 180 × 360 grid points with one degree resolution in θ and ϕ, respectively. The boundary conditions in longitude (ϕ) are periodic and for the poles ($\theta = 0, \pi$), we use the 180 degrees cross cells as the ghost cells for the first grid points from the pole.

2.3. Numerical Scheme

The advective term, $\mathbf{A}$, of equation (3) is discretized with a conservative two-stage Lax-Wendroff scheme. The diffusive term, $\mathbf{D}$, is added using an implicit time and central spatial discretization:

$$B_o^{n+1} = B_o^n + \mathbf{A}(B_o^n) + \mathbf{D}(B_o^{n+1}). \tag{8}$$

This linear system is solved using BiCGSTAB method (Van Der Vorst 1992).

If the diffusive term had been solved explicitly, the stability condition would allow a time step of the order of $\Delta t = 10^{-1}$ sec. The CFL stability condition for the advective term allowed a time step of the order of $\Delta t = 50$ sec. We applied filtering by averaging the first and last latitude grid rows; this filtering practically increased the size of the smallest cells and, as a result, the time step allowed by the CFL stability condition increased as well. The use of filtering and implicit scheme for the diffusive term allowed a time step of $\Delta t = 150$ sec, which resulted in a much faster convergence to steady state.

2.4. Initial Conditions

The initial distribution of the open magnetic flux was set using the Potential Field Source Surface extrapolation (Altschuler et al. 1977). The harmonic coefficients were taken from the Stanford WSO webpage (http://soi.stanford.edu/˜wso) and the field configuration was mapped to the solar surface. Although we use the observed field configuration, our investigation does not require the use of the real field and can instead use an idealized dipole configuration. Figure 1 shows the initial distribution of the magnetic flux for solar minimum (CR1908) and for solar maximum (CR1959).

3. Simulation Results

We simulated the transport of open magnetic flux on the surface of the Sun. For each iteration, in addition to advancing the solution for the magnetic flux density, we calculated the diffusion coefficient, κ, and the velocity, $\mathbf{w}$, which is a result of the reconnection process. Figure 2 shows the diffusion coefficient and the components of $\mathbf{w}$ for solar minimum and solar maximum initial conditions.

For solar minimum case, there is a preferred, large-scale, poleward gradient in κ, and $\mathbf{w}_\theta$ has a magnitude of $2 - 7\ m/s$, which is the same scale as the background meridional flow. The sign of $\mathbf{w}_\theta$ corresponds to latitude, meaning that negative flow is northward and positive flow is southward. We calculated $\mathbf{w}$ for solar minimum configurations with opposite polarities (CR1766, CR1908) and in both cases the flow was poleward. The magnitude of $\mathbf{w}_\phi$ is negligible compared to the magnitude of $\mathbf{w}_\theta$ in the case of solar minimum, and the magnitude of both components is very large close to the current sheet, due to the strong gradient in this region (as a result of the change in the sign of the magnetic field). However, in the Fisk model, reconnection does not occur in the current sheet, therefore, this region does not contribute to $\mathbf{w}$. In our simulation, although the magnitude of $\mathbf{w}$ in these regions is high, it always appears in pairs of positive and negative velocity in a relatively small area. As a result, the net effect of $\mathbf{w}$ is canceled in these regions, and we expect to see a significant effect only in large-scale regions with a preferred direction of $\mathbf{w}$.

In the case of solar maximum, the gradient of κ is isotropic and there are opposite components of $\mathbf{w}$ in most regions, so we expect the net effect of the reconnection processes to be smaller.

3.1. Solar Minimum Case

Figure 3 shows the steady state solution for solar minimum case. A steady state is achieved when the poleward motion is balanced by the equatorward diffusion, which is dominated by the uniform diffusive motion of the field line footpoints in the network lanes. For the case of uniform diffusion, a steady state is achieved after about 2 years (about 30 CR), and the 'Coronal holes boundary' is located at about 50 degrees from the poles. In the case of non-uniform diffusion a steady state is achieved after 2.5 years (about 45 CR) and the boundary moves poleward to about 35 degrees from the poles. Figure 4 shows the distribution of the magnetic field for the initial conditions and for the steady state with uniform and non-uniform diffusion coefficient at $\phi = 180°$. We found that in the solar minimum steady state solution, the meridional flow is enhanced by the magnetic reconnection processes. The enhancement is due to a large-scale preferred direction in the gradient of κ and in $\mathbf{w}_\theta$. During solar minimum conditions, the solar magnetic field is organized in a dipolar shape with two separate regions of open and closed magnetic field lines. Each of these regions has a typical rate of emergence of loops on the solar surface so the spatial change in the rate of emergence of loops is also directed poleward. All of the above lead us to the conclusion that the effect of diffusion due to reconnection processes is significant in a structured configuration of the solar magnetic field. This is mainly due to the fact that all the parameters, which control the behavior of the reconnection diffusion, in particular the rate of emergence of loops, have different characteristics in regions of open and closed magnetic field lines.

Since the meridional component of $\mathbf{w}$ depends on B_l, variations from cycle to cycle in the rate of emergence of new flux can cause changes in the solar surface meridional flow. As mentioned in Section 1, it has been suggested that variations in the solar surface meridional flow can explain the variations in the solar cycle. Our simulation showed that reconnection of open field lines with closed loops is an effective mechanism to modify the meridional flow.

3.2. Solar Maximum Case

Figure 5 shows the steady state solution for solar maximum conditions with uniform diffusion due to footpoints motion in the network lanes, and non-uniform diffusion, which also includes the effect of reconnection. It can be seen that the steady state solutions for the cases of uniform and non-uniform diffusion are very similar. The reason that the two cases are almost the same can be found in Fig 2. In the solar maximum configuration the solar magnetic field is unstructured. The gradient in κ is isotropic and, as a result, there is no preferred enhancement in the background flow. Solar maximum configuration results in local enhancements of the background flow. However, these local enhancements appear in pairs of opposite velocities so the large-scale net increase is very small.

Our simulation shows that the effect of diffusion due to magnetic reconnection is significant for the case of structured magnetic configuration (solar minimum conditions) and is small for the case of unstructured magnetic configuration (solar maximum conditions).

4. Discussion

We investigated the temporal evolution of the photospheric open magnetic flux for solar minimum configuration and solar maximum configuration. The study included the effect of diffusion of the open field lines due to magnetic reconnection, in addition to field line footpoints diffusion in the network lane and advective motion. Our two-dimensional simulation demonstrates that magnetic reconnection processes between open field lines and closed loops plays an important role in the transport of the open magnetic flux on the photosphere. We found that the effect of magnetic reconnection is significant for solar minimum conditions, but small for solar maximum conditions. In addition, we found that during solar minimum conditions, the flow is enhanced mainly poleward. Therefore, the effect of reconnection of open field lines with closed loops can be an effective mechanism to modify the solar surface meridional flow and cause variations over the solar cycle. Cameron and Hopkins (1998) found that large observed magnetic features tends to diffuse faster than small magnetic features. This is consistent with the idea that the displacement of an open field line as a result of reconnection is dependent on the size of the reconnected loop; in other words, the larger the loop, the larger the displacement. Snodgrass and Dailey (1995) mention that meridional flow is stronger in regions where the flux butterflies are steepest, which is consistent with the flow dependence on the spatial gradient of the emerging flux. Our simulation also demonstrates the importance of the large-scale effects and may raise the following question: Since all estimates of meridional flow result in an average flow pattern, is it possible that all the meridional motions are actually the same large-scale average motions due to the random walk of footpoints and the random magnetic reconnection events? If this is the case, then photospheric flux-transport models should be focused more on the investigation of random motions on the photosphere. Since equatorward flow in the deep Sun, which is required due to mass conservation considerations, has not been observed yet, it may be helpful to investigate the problem in terms of magnetic diffusion and random behavior of the deep Sun magnetic fields.

Our simulation shows the complexity of the solar magnetic field, and gives us physical insight into the importance of the convection zone, and the solar corona, as a single coupled system. We demonstrated that the distribution of the rate of emergence of flux on the photosphere can control the magnitude of the meridional flow and affect the variations from one solar cycle to another. However, the distribution of new emerging flux is dependent on internal processes, which can determine the variations, and these processes are not fully understood.

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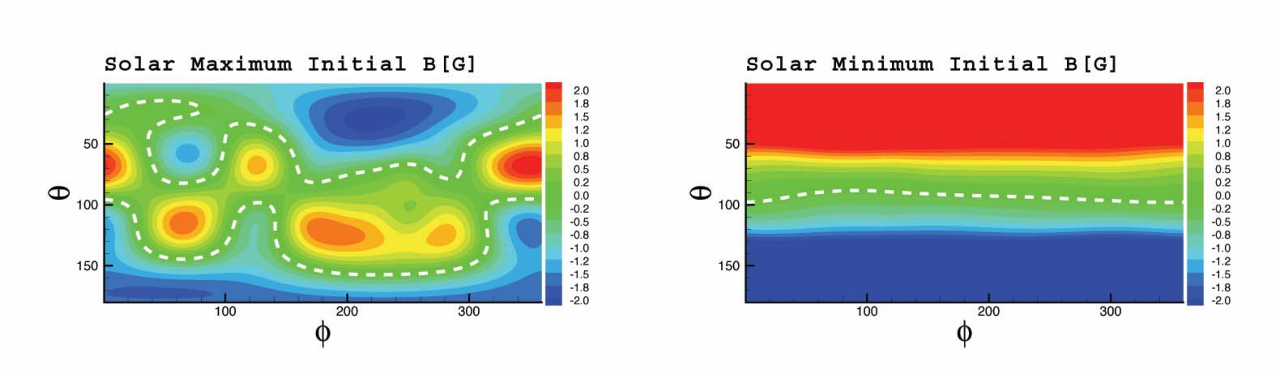

Fig. 1.— Initial distribution of the open magnetic flux (radial magnetic field) on the solar surface for solar maximum conditions (CR1959 - left) and for solar minimum conditions (CR1908 - right). The white dashed line marks the location of the current sheet. Figures 1a-1b are available in the electronic edition of the Journal. The printed edition contains only a sample.

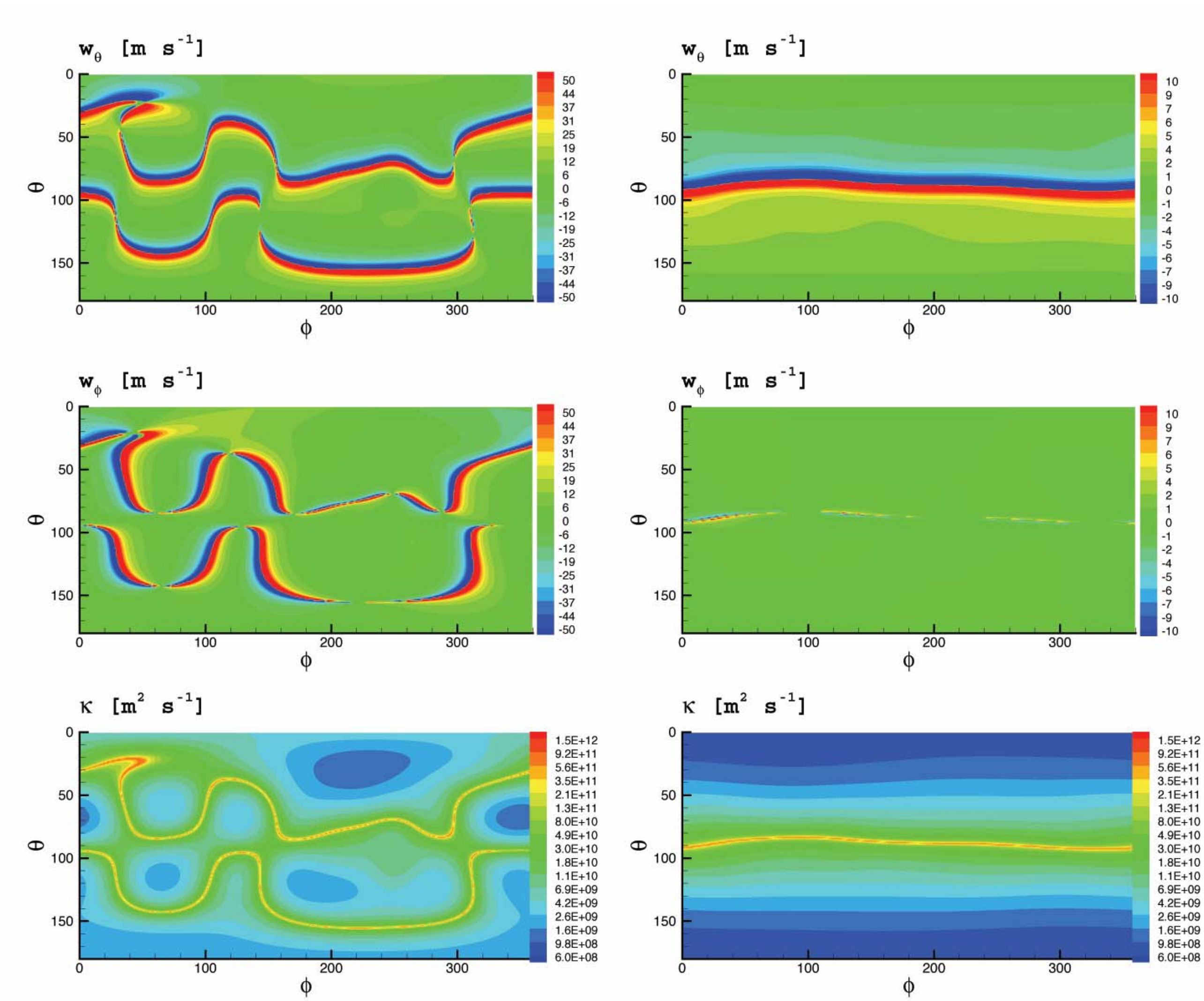

Fig. 2.— Meridional (top) and Azimuthal (middle) components of **w** and κ (bottom) for initial distribution of solar maximum (left) and solar minimum (right). Figures 2a-2f are available in the electronic edition of the Journal. The printed edition contains only a sample.

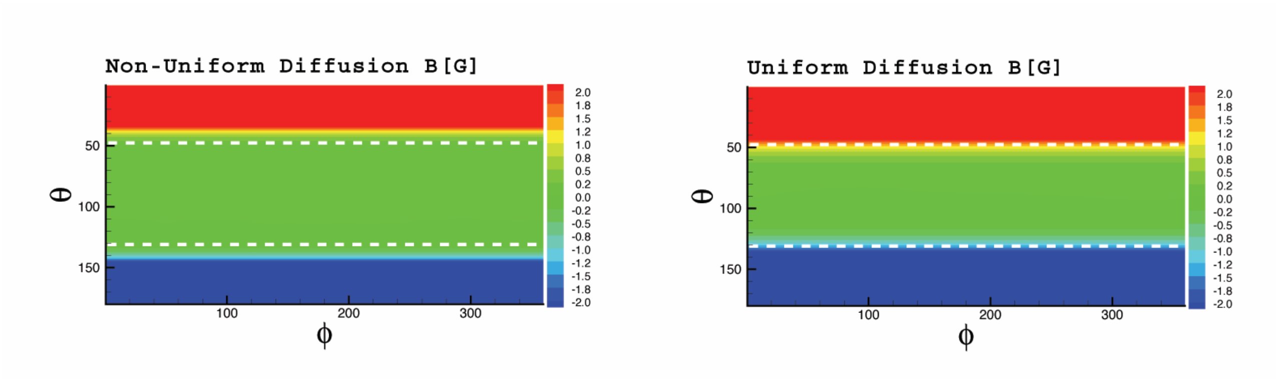

Fig. 3.— Steady-state solution for solar minimum conditions with non-uniform diffusion (left) and with uniform diffusion (right). The white dashed line marks the location of the boundaries of the coronal holes for the case of uniform diffusion. Figures 3a-3b are available in the electronic edition of the Journal. The printed edition contains only a sample.

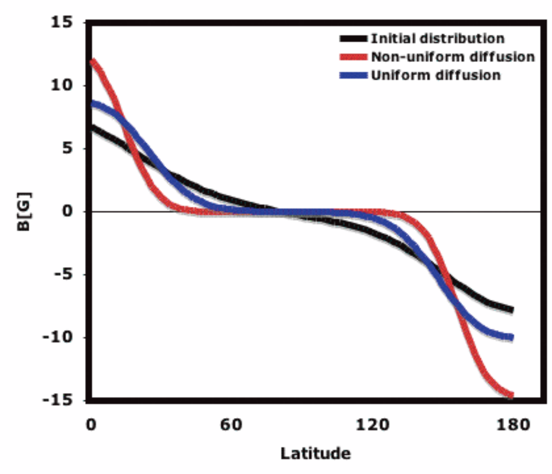

Fig. 4.— A latitudinal cut at $\phi = 180°$ for the solar minimum case at steady state. The black line represents the initial state, the blue line represents the steady state solution with uniform diffusion, and the red line represents the steady state solution with non-uniform diffusion.

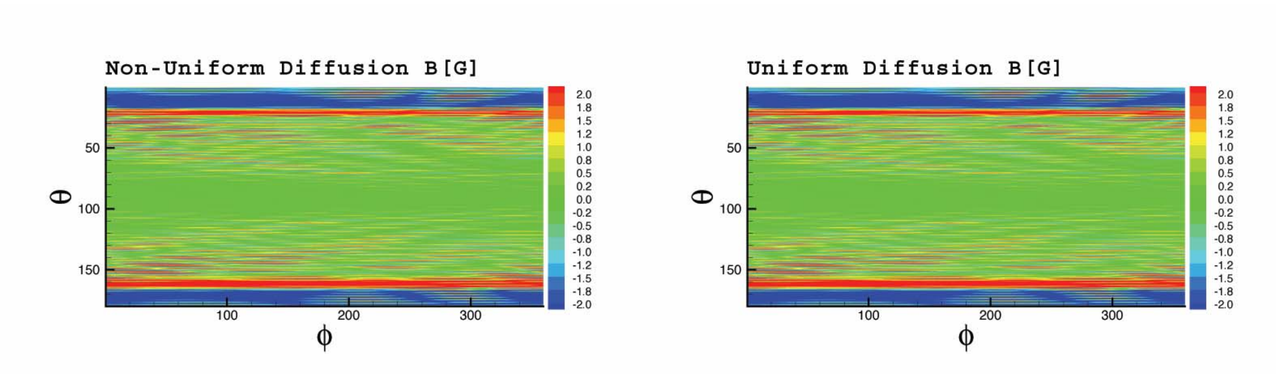

Fig. 5.— Steady-state solution for solar maximum conditions with non-uniform diffusion (left) and with uniform diffusion (right). Figures 5a-5b are available in the electronic edition of the Journal. The printed edition contains only a sample.